NUMBER OF PARTITIONS WITH PARTS OF THE FORM $pt + a$

Abstract.

Following the work of Ramanujan, Rademacher, and Hardy on proving an asymptotic formula for the number of integer partitions of a number, there have been similar efforts to find exact or asymptotic formulas for the number of partitions with parts $\pmod{m}$ belonging to a subset $A \subseteq \{0, 1, ..., m-1\}$. As a nice example, Livingood gave an exact formula for the number of partitions of n with parts of the form $pt \pm a$, with p a fixed prime:

$$p_a(n) = \sum_{p \nmid k,\rho,b} A_{k,b,\rho}(n) J_1\left(\frac{\pi\sqrt{(A-12pn)(B-12pb\rho)}}{3pk}\right) + \sum_{p|k,b} B_{k,b}(n) J_1\left(\frac{\pi\sqrt{A-12pn}}{3pk}\right).$$

where $A_{k,b,\rho}, B_{k,b}$ are Kloosterman sums and J_1 is a Bessel function. A more recent result of Laughlin et al gives a formula for the number of partitions with parts not divisible by integers r or s, which is represented by Bessel functions. These types of partitions are intrinsically interesting to study, but there is another motivation for doing so: the coefficients of Poincare series, which are a Maass Form, have the same form. This motivates one to imagine the generating functions of such partitions as a series of Poincare series. Poincare series play a key role in a few proofs of half-integral weight modular forms like Shimura-Shintani lifts.

In the proposed talk, we present a formula for the number of partitions with parts of the form $pt + a$ – which is different from the aforementioned problems, in that it applies to parts in a non-symmetric subset A mod m. We will also discuss the case where the parts are integer squares. Our main tool in proving these results we will use the Circle Method. The formulas that arise will involve incomplete Kloosterman sums and Bessel functions. Finally, we show that these results can also be obtained in a straightforward manner using the method of Meinardus.