Pentagonal Number Theorem and Zeroes of Riemann Zeta Function

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Abstract

We begin this talk by using Rademacher formula for $p(n)$ to state a generalization of partition functions of n to a real number x. In order to justify that this generalization is helpful, we prove Pentagonal Number Theory for the truncated version of Rademacher formula. In particular noting lth pentagonal number by G_l, and assuming RH, we prove that

$$\sum_{G_l<x} p(x-G_l) = O(\sqrt{p(x)}).$$

We use this equation to study the behaviour of Chebyshev function $\Psi(x)$. We prove that

$$\sum_{G_l<x} (-1)^l \Psi(e^{\frac{\pi}{6}\sqrt{24(x-G_l)-1}}) \left(\frac{1}{24(x-G_l)-1} - \frac{6}{\pi(24(x-G_l)-1)^{\frac{3}{2}}} \right) = O\left(\frac{e^{\frac{\pi(\frac{1}{2}+\delta)}{6}\sqrt{24x-1}}}{\sqrt{24x-1}} \right)$$

This fact gives us information about the distribution of zeros of the Riemann Zeta function, as we will discuss in this talk.